Álgebra grupo compleja de $\mathbb{Z}_3$.

Vamos a hacer la descomposición de Peirce de $A = \mathbb{C}\,\mathbb{z}_3$.

Tomamos $\mathbb{Z}_3 = \{1,\ \pi,\ \pi^2\}$ donde suponemos que $\pi^3 = 1$.

```
Unprotect[π];
```
⌊desprotege

```
e = x₀ + x₁ π + x₂ π²;
```

```
reglas = {π³ → 1, π⁴ → π};
```

```
Collect[e - Expand[e²] //. reglas, π]
```
⌊agrupa coefic··· ⌊expande factores

$$-x_0^2 + x_0 - 2\,x_1\,x_2 + \pi^2\left(-x_1^2 - 2\,x_0\,x_2 + x_2\right) + \pi\left(-x_2^2 - 2\,x_0\,x_1 + x_1\right)$$

```
Reduce[- x₀² + x₀ - 2 x₁ x₂ == 0 && - x₁² - 2 x₀ x₂ + x₂ == 0 && - x₂² - 2 x₀ x₁ + x₁ == 0]
```
⌊reduce

$$(x_2 = 0 \wedge x_1 = 0 \wedge (x_0 = 0 \vee x_0 = 1)) \bigvee$$

$$\left(\left(\left[x_2 = -\frac{1}{3} \bigvee x_2 = \frac{1}{3} \bigvee x_2 = -\frac{\sqrt[3]{-1}}{3} \bigvee x_2 = \frac{\sqrt[3]{-1}}{3} \bigvee x_2 = -\frac{1}{3}(-1)^{2/3} \bigvee x_2 = \frac{1}{3}(-1)^{2/3}\right] \bigwedge\right.\right.$$

$$\left.\left. x_1 = 81\,x_2^5 \bigwedge x_0 = \frac{1}{2}\left(1 - 9\,x_2^3\right)\right)\right)$$

Aparte de las soluciones e=0 y e=1 que siempre tenemos, también están:

```
x₂ = ϵ / 3; x₁ = 81 x₂⁵; x₀ = 1/2 (1 - 9 x₂³);
```

```
e //. ϵ → 1
```

$$\frac{1}{3} + \frac{\pi}{3} + \frac{\pi^2}{3}$$

```
e //. ϵ → -1
```

$$\frac{2}{3} - \frac{\pi}{3} - \frac{\pi^2}{3}$$

```
x₂ = ϵ/3 Exp[i π / 3]; x₁ = 81 x₂⁵; x₀ = 1/2 (1 - 9 x₂³);
```
⌊exponencial

```
e //. ϵ → 1
```

$$\frac{2}{3} + \frac{1}{3}\,e^{-\frac{i\pi}{3}}\,\pi + \frac{1}{3}\,e^{\frac{i\pi}{3}}\,\pi^2$$

```
e //. ϵ → -1
```

$$\frac{1}{3} - \frac{1}{3}\,e^{-\frac{i\pi}{3}}\,\pi - \frac{1}{3}\,e^{\frac{i\pi}{3}}\,\pi^2$$

```
x₂ = ε/3 Exp[-i π / 3]; x₁ = 81 x₂⁵; x₀ = 1/2 (1 - 9 x₂³);
      exponencial
```

```
e //. ε → 1
```

$$\frac{2}{3} + \frac{1}{3} e^{\frac{i\pi}{3}} \pi + \frac{1}{3} e^{-\frac{i\pi}{3}} \pi^2$$

```
e //. ε → -1
```

$$\frac{1}{3} - \frac{1}{3} e^{\frac{i\pi}{3}} \pi - \frac{1}{3} e^{-\frac{i\pi}{3}} \pi^2$$

```
e₁ = 1/3 + π/3 + π²/3;
e₂ = 1/3 - 1/3 e^{-iπ/3} π - 1/3 e^{iπ/3} π²;
e₃ = 1/3 - 1/3 e^{iπ/3} π - 1/3 e^{-iπ/3} π²;
```

```
Simplify[{e₁ + e₂ + e₃ - 1, e₁² - e₁, e₂² - e₂, e₃² - e₃}] //. reglas
simplifica
```

{0, 0, 0, 0}

```
Simplify[Expand[e₁ e₂] //. reglas]
simplifica   expande factores
```

0

```
Simplify[Expand[e₁ e₃] //. reglas]
simplifica   expande factores
```

0

```
Simplify[Expand[e₂ e₃] //. reglas]
simplifica   expande factores
```

0

```
Expand[(x + y π + z π²) e₁ == (x + y + z) e₁] //. reglas
expande factores
```

True

```
Expand[(x + y π + z π²) e₂ == (x - Exp[i π / 3] y - Exp[-i π / 3] z) e₂] //. reglas //
expande factores                        exponencial              exponencial
  Simplify
  simplifica
```

True

```
Expand[(x + y π + z π²) e₃ == (x + Exp[2 i π / 3] y - Exp[i π / 3] z) e₃] //. reglas //
expande factores                        exponencial              exponencial
  Simplify
  simplifica
```

True