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THE FREUDENTHAL-SPRINGER-TITS CONSTRUCTIONS OF EXCEPTIONAL JORDAN ALGEBRAS

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Constructions of 27-dimensional exceptional simple Jordan algebras have been given by H. Freudenthal [2], [3], T. A. Springer [8], and J. Tits [9]. In the first two approaches the cubic generic norm form plays a central role, with applications to projective geometry and algebraic groups; the third approach gives a simple method for constructing all exceptional simple algebras. The constructions are limited to fields of characteristic $\neq 2$ as usual for Jordan algebras defined in terms of a bilinear multiplication, and in order to polarize the cubic norm form the characteristic must be $\neq 3$.

Recently a definition of Jordan algebras has been proposed [5] which is based on a cubic composition involving U-operators. A *unital Jordan algebra* over a commutative associative ring Φ is a triple $\mathfrak{J}=(\mathfrak{X}, U, 1)$ where $\mathfrak{X}$ is a Φ-module, U a quadratic mapping $x \to U_x$ of $\mathfrak{X}$ into $\mathrm{Hom}_\Phi(\mathfrak{X}, \mathfrak{X})$, and 1 an element of $\mathfrak{X}$ satisfying the axioms

(1) $U_1 = 1$,

(2) $U_{U(x)y} = U_x U_y U_x$,

(3) $U_x\{yxz\} = \{U_x yzx\}$.

($\{xyz\} = U_{x,z}y$ for $U_{x,z} = U_{x+z} - U_x - U_z$), and such that these hold under all scalar extensions (equivalently, the axioms can be linearized). For fields of characteristic $\neq 2$ this is equivalent to the usual definition of Jordan algebras (with $U_x = 2L_x^2 - L_{x^2}$).

In this paper the constructions of Freudenthal, Springer, and Tits will be derived as special cases of one general construction valid for all characteristics. The basic axioms used go back to [2]. It was Professor Springer who first pointed out that this approach could be carried out for arbitrary characteristics, and the author is indebted to him for suggesting the explicit formula for the Jordan structure.

1. **The general construction.** Let $\mathfrak{X}$ be a module over a commutative associative ring Φ. We assume we are given (i) a cubic form N on $\mathfrak{X}$ with values in Φ (so N is homogeneous of degree 3 and $N(x+\lambda y) = N(x) + \lambda\, \partial_y N|_x + \lambda^2\, \partial_x N|_y + \lambda^3 N(y)$ where the differential $\partial_y N|_x$ of N at x in the direction y is linear in y and quadratic in x), (ii) a quadratic mapping $x \to x^\#$ in $\mathfrak{X}$, and (iii) a basepoint $c \in \mathfrak{X}$ related by

(4) $x^{\#\#} = N(x)x$ ("adjoint identity"),

(5) $N(c) = 1$ ("basepoint identity"),

Received by the editors May 28, 1968.