Clase del 30-X-20. Teoría de representaciones.

Producto tensorial de esp. vect. Fijemos un cuerpo K y dos K-esp. vect. E y F

$L^2(E\times F;K)=\{B: E\times F\longrightarrow K \;/\; B \text{ es bilineal}\}$. Es un K-esp. vectorial.

$\forall B_1,B_2\in L^2(E\times F;K)$, $\quad B_1+B_2: \begin{array}{l} E\times F\longrightarrow K\\ (e,f)\longmapsto B_1(e,f)+B_2(e,f)\end{array}$ $\Big\}$ $(L^2(E\times F;K),+)$ G° Abel.

$\left.\begin{array}{l} K\times L^2(E\times F;K)\longrightarrow L^2(E\times F;K)\\ (\alpha,B)\longmapsto \alpha B: \begin{array}{l} E\times F\longrightarrow K\\ (e,f)\longmapsto \alpha B(e,f)\end{array}\end{array}\right\}$ $(L^2(E\times F;K),+,\cdot)$ es un K-esp. vect.

Dado un e.v. E sobre K, $\dim(E)=n$ finita, $\boxed{B=\{e_1,\dots,e_n\} \text{ base de } E}$

$E^*:=\hom(E,K)=\{T:E\to K \;/\; T \text{ lineal}\}$ dual de E

$\underline{\underline{e_i\in B}}$, $\quad \left.\begin{array}{l}\hat{e}_i: E\longrightarrow K\\ \quad e_i\longmapsto 1\\ \quad e_j\longmapsto 0 \text{ si } j\neq i\end{array}\right\}$ $\quad \begin{array}{l}\hat{e}_i(e_j)=\delta_{ij} \text{ Kronecker}\\ \hat{e}_i\in E^*\end{array}$ $\quad \{\hat{e}_i\}_{i=1}^n=\{\hat{e}_1,\dots,\hat{e}_n\}$ $\underline{\underline{\text{base de } E^*}}$

$\dim(E^*)=\dim(E)$

Dados $e \in E$, $f \in F$ definimos $\boxed{\begin{array}{l} e\otimes f : E^* \times F^* \longrightarrow K \\ (\alpha, \beta) \longmapsto \alpha(e)\beta(f) \end{array}}$ ¿$e\otimes f \in L^2(E^* \times F^*; K)$?

$$E \xrightarrow{\alpha} K \qquad e \longmapsto \alpha(e)$$

$$F \xrightarrow{\beta} K \qquad f \longmapsto \beta(f)$$

$$\begin{cases} (e\otimes f)(\alpha+\alpha', \beta) \overset{1º}{=} (e\otimes f)(\alpha, \beta) + (e\otimes f)(\alpha', \beta) & \forall \alpha, \alpha' \ \forall \beta \\ (e\otimes f)(k\alpha, \beta) = k(e\otimes f)(\alpha, \beta), & \forall k \in K, \ \forall \alpha \in E^*, \forall \beta \in F^* \\ (e\otimes f)(\alpha, \beta+\beta') = (e\otimes f)(\alpha, \beta) + (e\otimes f)(\alpha, \beta') & \forall \alpha, \forall \beta, \beta' \\ (e\otimes f)(\alpha, k\beta) = k(e\otimes f)(\alpha, \beta) \end{cases}$$

$$\boxed{(e\otimes f)(\alpha+\alpha', \beta) \overset{def}{:=} (\alpha+\alpha')(e)\beta(f) = [\alpha(e)+\alpha'(e)]\beta(f) = \alpha(e)\beta(f) + \alpha'(e)\beta(f) = (e\otimes f)(\alpha, \beta) + (e\otimes f)(\alpha', \beta)}$$

$$e\otimes f \in L^2(E^* \times F^*; K)$$

$$E\otimes F \overset{def}{:=} \left\{ \sum_i k_i (e_i \otimes f_i) \ / \ k_i \in K, \ e_i \in E, \ f_i \in F \right\} \subset L^2(E^* \times F^*; K)$$

La definición es válida para E, F de dim arbitraria.

Si dim(E), dim(F) son finitas $E\otimes F = L^2(E^* \times F^*; K)$

Propiedades del prod. tensorial

1) Sup. E, F de dim finita, $\boxed{B_E = \{e_i\}_{i=1}^{n} \text{ base de } E}$; $B_F = \{f_j\}_{j=1}^{k}$ base de F

$\boxed{\{e_i \otimes f_j\}_{i=1, j=1}^{n\ k} \text{ es una base de } E \otimes F}$ ← $\boxed{\dim(E \otimes F) = \dim(E)\dim(F)}$

1.a) L.I. $\underbrace{\boxed{\sum_{i,j} k_{ij}(e_i \otimes f_j) = 0}}$ $\overset{?}{\Longrightarrow}$ $\boxed{k_{ij} = 0 \quad \forall i\ \forall j}$

$e_i \otimes f_j$; $\underbrace{E^* \times F^*} \to K$, tomamos $\hat{e}_p \in E^*$, $\hat{f}_q \in F^*$ $\hat{e}_p(e_r) = \delta_{pr}$

$\hat{f}_q(f_t) = \delta_{qt}$

$$\left[0 = \underbrace{\sum_{ij} k_{ij}(e_i \otimes f_j)(\hat{e}_p, \hat{f}_q)}_{p,q \text{ cualquiera}} = \sum_{ij} k_{ij}\, \hat{e}_p(e_i)\hat{f}_q(f_j) = \right.$$

$$= \sum_{ij} k_{ij}\, \delta_{pi}\, \delta_{qj} = \boxed{k_{pq}}$$

$\boxed{\text{Todos los } k_{pq} = 0}$

1.b) Como $E \otimes F$ consiste en comb. lineales de elementos $e \otimes f$ con $e \in E$, $f \in F$

$\forall e = \sum_i x_i e_i$ $x_i \in K$, $\forall f = \sum_j y_j f_j$ $y_j \in K$

$$\boxed{e \otimes f} = \left(\sum_i x_i e_i\right) \otimes \left(\sum_j y_j f_j\right) = \boxed{\sum_{ij} x_i y_j\, e_i \otimes f_j}$$ por las prop. del $\otimes$

2) E, F dimensión finita $\dim(E) = n$, $\dim(F) = k$

$$E \otimes F \subset L^2(E^* \times F^*; K)$$

$$\dim(E \otimes F) = nk$$

$$\dim(L^2(E^* \times F^*; K)) = nk$$

$$\boxed{E \otimes F := L^2(E^* \times F^*; K)}$$

$\forall T \in L^2(E^* \times F^*; K)$

$\{\hat{e}_i\}_{i=1}^{n}$ base de E^*

$\{\hat{f}_j\}_{j=1}^{k}$ " " F^*

$L^2(E^* \times F^*; K) \longrightarrow M_{n,k}(K)$ = esp. de matrices $n \times k$ con coef en K

$$T \longmapsto \begin{pmatrix} T(\hat{e}_1, \hat{f}_1) & \cdots & T(\hat{e}_1, \hat{f}_k) \\ T(\hat{e}_2, \hat{f}_1) & \cdots & T(\hat{e}_2, \hat{f}_k) \\ T(\hat{e}_n, \hat{f}_1) & \cdots & T(\hat{e}_n, \hat{f}_k) \end{pmatrix} \in M_{n,k}(K)$$

Isom. de esp. vect.

$$\dim(M_{n,k}(K)) = nk$$

<u>Propiedad</u> <u>universal</u> <u>de</u> <u>$E \otimes F$</u>:

$E \times F \xrightarrow{\tau} E \otimes F$ Se llama la aplic. tensorial

$(e, f) \longmapsto e \otimes f$ τ es aplicación bilineal

$E \times F \xrightarrow{T} V$ T cualq. aplic. bilineal

$\exists!\ \theta : E \otimes F \longrightarrow V$ lineal tal que $\theta\tau = T$

$\{e_i\}_{i=1}^{n}$ base de E ; $\{f_j\}_{j=1}^{k}$ base de F

$$\begin{tikzcd} E \times F \arrow[r, "\tau"] \arrow[dr, "T"'] & E \otimes F \arrow[d, dashed, "\theta"] \\ & V \end{tikzcd}$$

$\theta(e_i \otimes f_j) = \theta\tau(e_i, f_j) = T(e_i, f_j)$

Defino $\theta(e_i \otimes f_j) := T(e_i, f_j)$

θ se extiende por linealidad.

$\theta\tau = T$

3) $E \otimes F \overset{(a)}{\cong} F \otimes E$ isom. de esp. vect.

$$\begin{tikzcd} E \times F \arrow[r, "\tau"] \arrow[d, "ex"'] & E \otimes F \arrow[d, dashed, "\theta"] \\ F \times E \arrow[r, "\tau'"] & F \otimes E \end{tikzcd}$$

$(e,f) \mapsto (f,e) \mapsto f \otimes e$

$ex =$ intercam.

$$\boxed{\theta(e \otimes f) = T(e,f) = (\tau' \circ ex)(e,f) = \tau'(f,e) = f \otimes e}$$

$T = \tau' \circ ex$ es bilineal

Análog. $F \otimes E \xrightarrow{\theta'} E \otimes F$ lineal.

$\theta'(f \otimes e) = e \otimes f$

$\theta' \circ \theta = 1_{E \otimes F}$

$\theta \circ \theta' = 1_{F \otimes E}$

4) $E \otimes (F \otimes G) \overset{(b)}{\cong} (E \otimes F) \otimes G$ isom. de esp. vect.

5) K cuerpo base, E un K-esp. vect.
K es K-esp. vect.

$E \otimes K \overset{(c)}{\cong} E$ isom. de esp. vect.

Dem.:

$$\begin{tikzcd} E \times K \arrow[r, "\tau"] \arrow[dr, "T"'] & E \otimes K \arrow[d, dashed, "\theta"] \\ & E \end{tikzcd}$$

$(e,k) \mapsto ke$

T es bilineal.

$\exists \theta$ lineal $\theta \tau = T$, $\theta(e \otimes k) = T(e,k) = ke$

$E \xrightarrow{\theta'} E \otimes K$

$e \longmapsto e \otimes 1$

$\boxed{\theta' \theta(e \otimes k)} = \theta'(ke) = (ke) \otimes 1 = k(e \otimes 1) = e \otimes (k \cdot 1) = \boxed{e \otimes k}$

$\boxed{\theta \theta'(e)} = \theta(e \otimes 1) = \boxed{e}$

$\begin{cases} \theta' \theta = 1_{E \otimes K} \\ \theta \theta' = 1_E \end{cases}$

6) $\left.\begin{array}{l} f: V_1 \longrightarrow V_2 \quad \text{aplic. lineal} \\ g: W_1 \longrightarrow W_2 \quad \text{"} \quad \text{"} \end{array}\right\}$ $\quad$ $\left.\begin{array}{l} f\otimes g: V_1\otimes W_1 \longrightarrow V_2\otimes W_2 \\ \qquad x\otimes y \longmapsto f(x)\otimes g(y) \end{array}\right\}$ aplic. lineal

$$V_1 \xrightarrow{f} V_2 \xrightarrow{h} V_3$$

$$W_1 \xrightarrow[g]{} W_2 \xrightarrow[k]{} W_3$$

$$\begin{tikzcd} V_1\otimes W_1 \arrow[r, "f\otimes g"] \arrow[rr, bend right, "(h\circ f)\otimes(k\circ g)"'] & V_2\otimes_k W_2 \arrow[r, "h\otimes k"] & V_3\otimes_k W_3 \end{tikzcd}$$

$$\boxed{(h\otimes k)\circ(f\otimes g) = (h\circ f)\otimes(k\circ g)}$$

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7) A, B dos K-álgebras, (K cuerpo) $\qquad A\otimes B$ es un K-esp. vect.

$$(A\otimes B)\times(A\otimes B) \longrightarrow A\otimes B$$

$(a_1\otimes b_1)(a_2\otimes b_2) := (a_1a_2)\otimes(b_1b_2)$ dota a $\boxed{A\otimes B \text{ de estructura de } K\text{-álgebra}}$

Caso particular. $\quad K$ cuerpo base, $\quad K\subset F$ cuerpo extensión de K (o sea K subcuerpo de F)

F es de forma natural un K-esp. vectorial.

Si $\boxed{A \text{ es una } K\text{-álgebra}}$ $\qquad A\otimes_K F$ es una K-álgebra.

$F\times(A\otimes_K F) \longrightarrow A\otimes_K F$, $\quad \lambda\in F, \mu\in F,$ $\quad \boxed{\lambda\cdot(a\otimes\mu) := a\otimes\lambda\mu}$

dota a $A\otimes F$ de estruct. de F-álg.

Originalmente: A solo es álg. sobre K.

$A_F := A \otimes_K F$ "extensión por escalares" de A al cuerpo F.

$\boxed{A \text{, tiene la base } \{e_i\}_{i=1}^{n} \overset{?}{\Longrightarrow} \{e_i \otimes 1\}_{i=1}^{n} \text{ es una base de } A_F \text{ como e.v. sobre } F}$

$\dim_K(A) = \dim_F(A_F)$

$$\boxed{\begin{array}{l} e_{ij}\,e_{kl} = 0 \quad \text{si } j \neq k \\ e_{ij}\,e_{jk} = e_{ik} \end{array}}$$

$A = M_2(\mathbb{R})$, $\quad B = \{e_{11}, e_{12}, e_{21}, e_{22}\}$

$e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $e_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$e_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ base canónica de $M_2(\mathbb{R})$

$\mathbb{R} \subset \mathbb{C}$. $\boxed{A_{\mathbb{C}} = A \otimes_{\mathbb{R}} \mathbb{C}}$ una base $B_{\mathbb{C}} = \{e_{11} \otimes 1, e_{12} \otimes 1, e_{21} \otimes 1, e_{22} \otimes 1\}$

	e_{11}	e_{12}	e_{21}	e_{22}
e_{11}	e_{11}	e_{12}	0	0
e_{12}	0	0	e_{11}	e_{12}
e_{21}	e_{21}	e_{22}	0	0
e_{22}	0	0	e_{21}	e_{22}

$(e_{11} \otimes 1)(e_{12} \otimes 1) = e_{12} \otimes 1$

$0 \otimes x = (a - a) \otimes x = a \otimes x - a \otimes x = 0$

$x \otimes 0 = 0$

	$e_{11}\otimes 1$	$e_{12}\otimes 1$	$e_{21}\otimes 1$	$e_{22}\otimes 1$
$e_{11}\otimes 1$	$e_{11}\otimes 1$	$e_{12}\otimes 1$	0	0
$e_{12}\otimes 1$	0	0	$e_{11}\otimes 1$	$e_{12}\otimes 1$
$e_{21}\otimes 1$	$e_{21}\otimes 1$	$e_{22}\otimes 1$	0	0
$e_{22}\otimes 1$	0	0	$e_{21}\otimes 1$	$e_{22}\otimes 1$

Caso general

A, $\{\underline{e_i}\}_{i=1}^n$ base

álgebra sobre K

$$\underset{\smile}{e_i}\,\underset{\smile}{e_j} = \sum_k c_{ijk}\, \underset{\smile}{e_k}$$

donde $\boxed{c_{ijk} \in K}$

c_{ijk} = coef. k-ésimo de $e_i e_j$ en la base.

c_{ijk} = entradas de la "tabla" de multiplicación.

A_F, $\{\underline{\underline{e_i \otimes 1}}\}_{i=1}^n$

álgebra sobre F

$$(e_i \otimes 1)(e_j \otimes 1) := (e_i e_j) \otimes 1 = \left(\sum_k c_{ijk}\, e_k\right) \otimes 1 = \sum_k \boxed{c_{ijk}}\,(e_k \otimes 1)$$

coef de $(e_i \otimes 1)(e_j \otimes 1)$ en la base $\{e_i \otimes 1\}$ es el mismo c_{ijk}.

Las const. de estruct. son las mismas que antes.

Caso particular. K cuerpo, $K \subset F$ = clausura algebraica de K

A sobre K, $\quad$ A_F sobre F

En lo que atañe a T. de repr.

K cuerpo, A una K-álgebra.

$\rightarrow$ $r_1 : A \xrightarrow{\text{repr.}} \mathrm{End}_K(V_1)$ es un hom. de K-álg.

$r_2 : A \longrightarrow \mathrm{End}_K(V_2)$ " " " "

$r_1(a) : V_1 \to V_1$

$r_2(a) : V_2 \to V_2$

Se puede construir: $r_3 : A \longrightarrow \mathrm{End}_K(V_1 \otimes V_2)$

$a \longmapsto r_3(a) : V_1 \otimes V_2 \xrightarrow{r_1(a)\otimes r_2(a)} V_1 \otimes V_2$

$\boxed{x \otimes y \longmapsto r_1(a)(x) \otimes r_2(a)(y)}$

$\boxed{r_3(a) := r_1(a) \otimes r_2(a)}$

¿Dem. que r_3 es hom. de K-álg?

$\boxed{17:45}$

$r_1 \oplus r_2 : A \longrightarrow \mathrm{End}_K(V_1 \otimes V_2)$

Descomposición de Peirce. Sea A una K-álgebra (donde K es un cuerpo), $1 \in A$, $\boxed{1 = e_1 + \cdots + e_n}$ donde $\boxed{e_i^2 = e_i}$, $e_i e_j = 0$ si $i \neq j$ (los e_i se dice que forman un sist. de idem, ortogonales, complet.)

Ejemplo. $A = M_n(K)$ $\quad e_i = \begin{pmatrix} 0 & \cdots & 0 & 1 & 0 & 0 \\ 0 & & & 0 & \cdots & 0 \end{pmatrix}$ $\quad$ (i,i) es 1, $(i,j) = 0$ si $i \neq j$, (j,j) 0 si $j \neq i$.

$$\left.\begin{array}{l} e_i^2 = e_i \\ e_i e_j = 0 \quad i \neq j \\ \sum_{i=1}^n e_i = Id \end{array}\right\}$$

Nos situamos en ese contexto. A un K-álg. $1 \in A$. $\{e_1, \ldots, e_n\}$ sist. de idemp ortog. completo.

$$A = \bigoplus_{i,j=1}^n A_{ij} \quad \text{donde cada} \quad A_{ij} = \{x \in A : e_i x = x = x e_j\} \quad (i,j = 1 \ldots n)$$

$$\boxed{A_{ij} = e_i A e_j} = \{e_i x e_j : x \in A\}$$

Cada A_{ij} es un subespacio vect. de A;

Dem. $x \in A_{ij} \Longleftrightarrow e_i x = x = x e_j \Longleftrightarrow \boxed{x = e_i x e_j} \Longleftrightarrow x \in e_i A e_j$

$$\boxed{A_{ij} = e_i A e_j}$$

Subespacios de Peirce

Veamos que $\boxed{A = \bigoplus_{i,j=1}^n A_{ij}}$

Teorema $A = \bigoplus_{i,j=1}^{n} A_{ij}$ DESCOMPOSICIÓN DE PEIRCE

Dem: $A = \sum_{i,j=1}^{n} A_{ij}$, $\forall a \in A$, $a \overset{?}{=} \sum_{i,j=1}^{n} a_{ij}$ donde $a_{ij} \in A_{ij} = e_i A e_j$

a arbitrario, $a = 1 \cdot a = \left(\sum_{i=1}^{n} e_i\right) a = \sum_{i=1}^{n} e_i a = \sum_{i=1}^{n} e_i a 1 = \sum_{i=1}^{n} e_i \left(a \sum_{j=1}^{n} e_j\right) = \sum_{i,j=1}^{n} e_i a e_j \in \sum_{i,j=1}^{n} A_{ij}$

$A = \sum_{i,j=1}^{n} A_{ij}$. La suma en realidad es directa.

$$\left(\sum_{i,j=1}^{n} x_{ij} = 0\right) \overset{?}{\Longrightarrow} \left(\left(x_{ij} = 0 \quad \forall i,j\right)\right)$$

Si $0 = \sum_{i,j=1}^{n} x_{ij}$; $0 = \left(\sum_{i,j=1}^{n} x_{ij}\right) e_p = \sum_{i=1}^{n} \boxed{x_{ip}}$;

p arbitrario

$0 = e_q \left(\sum_{i=1}^{n} x_{ip}\right)$ con q arbitr.

$\boxed{0 = x_{qp}}$

$e_q x_{ip} = 0$ si $q \neq i$

$e_q x_{qp} = x_{qp}$

A_{qp}

$x_{ij} e_p$ cuando $\boxed{j \neq p}$; $x_{ij} e_p \in e_i A \overbrace{e_j e_p}^{0} = 0$

$x_{ip} e_p = x_{ip} e_p e_p = x_{ip} e_p = x_{ip}$

$A_{ij} = \{a \in A : e_i a = a = a e_j\}$

Ejemplo $A = M_n(K)$ $\{e_1, e_2, \dots, e_n\}$ $e_i := \mathrm{diag}(0 \dots 0 \overset{i}{1} 0 \dots 0)$ ← son matrices elementales.

E_{ij} = matriz que tiene un 1 en el cruce de la fila i-ésima con la columna j-ésima.

$\{E_{ij} : i, j = 1 \dots n\}$ $E_{11} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ & & 0 & \end{pmatrix}$ $E_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & & & & \end{pmatrix}$

El conj. tiene cardinal n^2

Las E_{ij} se llaman matrices-elementales.

$(a_{ij})_{i,j=1}^n \in A$ $(a_{ij})_{i,j=1}^n = \sum_{i,j=1}^n a_{ij} E_{ij}$. $\{E_{ij} : i,j = 1 \dots n\}$ es sist. de generadores

Además es un conj. L.I. si $\sum_{i,j=1}^n \lambda_{ij} E_{ij} = 0 \implies \lambda_{ij} = 0$. $\underline{\{E_{ij} : i,j = 1 \dots n\}}$ base de A

El conjunto $\{e_1, e_2, \dots, e_n\}$ es un sist. de idem. ortog. que suman Id.

$$e_i = E_{ii} \qquad \begin{cases} e_i^2 = e_i \\ e_i e_j = 0 \quad \text{si } i \neq j \\ \sum e_i = Id \end{cases}$$

$$\boxed{\begin{array}{l} E_{ij} E_{jk} = E_{ik} \\ E_{ij} E_{kl} = 0 \quad \text{si } j \neq k \end{array}}$$

$$A = M_n(K) = \bigoplus_{i,j=1}^n \boxed{e_i A e_j} = \bigoplus_{i,j=1}^n \boxed{A_{ij}}$$

$A_{ij} = e_i A e_j = K E_{ij}$ = son los espacios de Peirce

$$e_i \Big(\sum_{pq} \lambda_{pq} E_{pq}\Big) e_j = E_{ii} \Big(\sum_{pq} \lambda_{pq} E_{pq}\Big) E_{jj} = \lambda_{ij} E_{ij}$$

__Problema__

Tomemos $\mathbb{H} = \{\lambda_0 1 + \lambda_1 i + \lambda_2 j + \lambda_3 k \;:\; \lambda_n \in \mathbb{R}\}$.

$\boxed{i^2 = j^2 = k^2 = ijk = -1}$

$\mathbb{H}$ es un $\mathbb{R}$-álg. $\dim_{\mathbb{R}}(\mathbb{H}) = 4$, $\{1, i, j, k\}$

$\mathbb{C}$ es un $\mathbb{R}$-álg. $\dim_{\mathbb{R}}(\mathbb{C}) = 2$, $\{1, i\}$

$$\boxed{\mathbb{H} \otimes_{\mathbb{R}} \mathbb{C} \overset{?}{\cong} M_2(\mathbb{C})}$$

$i \to j \to k \to i$

$\dim(\mathbb{H} \otimes_{\mathbb{R}} \mathbb{C}) = \dim_{\mathbb{R}}(\mathbb{H}) \dim_{\mathbb{R}}(\mathbb{C}) = 4 \cdot 2 = 8$

$\dim_{\mathbb{R}}(M_2(\mathbb{C})) = 8$ $\qquad$ $\dim_{\mathbb{C}}(M_2(\mathbb{C})) = 4$

Descomp. de Peirce de $M_2(\mathbb{C})$ $\quad$ s. de idemp. $\{e_1, e_2\}$ ortog. complet.

$e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$,

$M_2(\mathbb{C}) = K E_{11} \oplus K E_{12} \oplus K E_{21} \oplus K E_{22}$

$E_{11} = e_1$, $E_{22} = e_2$

$E_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $E_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

Localizar idempotentes en $\mathbb{H} \otimes_{\mathbb{R}} \mathbb{C}$

En general. A álgebra, si $\boxed{\exists x \in A : x^2 = 1}$, $\mathrm{char}(K) \neq 2$

$\boxed{\tfrac{1}{2} \in K}$

$e = \frac{1}{2}(1+x)$; $e^2 = \frac{1}{4}(1 + \underset{\substack{\shortparallel \\ 1}}{x^2} + 2x) = \frac{1}{4}(2 + 2x) = \frac{1}{2}(1+x) = e$

$f = \frac{1}{2}(1-x)$; $f^2 = \frac{1}{4}(1 + x^2 - 2x) = \frac{1}{4}(2 - 2x) = \frac{1}{2}(1-x) = f$

$\boxed{e + f = 1}$; $ef = \frac{1}{4}(1+x)(1-x) = \frac{1}{4}(1-x^2) = \frac{1}{4}(1-1) = 0$

$(i \otimes i)(i \otimes i) = i^2 \otimes i^2 = 1 \otimes 1 = 1$

Luego $e_1 := \frac{1}{2}(1 + i\otimes i)$; $e_2 := \frac{1}{2}(1 - i\otimes i)$ Dos idemp. ortogonales de $e_1 + e_2 = 1$ OJO: $1 = 1\otimes 1$ en $\mathbb{H}\otimes\mathbb{C}$

$A = \mathbb{H}\otimes\mathbb{C}$ Descomp. de Peirce de A relativa a $\{e_1, e_2\}$

$A = e_1Ae_1 \oplus e_1Ae_2 \oplus e_2Ae_1 \oplus e_2Ae_2$

$e_1Ae_1 \overset{?}{=} \mathbb{C}\,\frac{1}{2}(1+i\otimes i) = \mathbb{C}(1+i\otimes i)$

$e_2Ae_2 \overset{?}{=} \dots\dots = \mathbb{C}(1-i\otimes i)$

$e_1Ae_2 = \mathbb{C}(\,?\,)$

$\uparrow \Uparrow \uparrow$

$\boxed{\text{Marta: } 16:00 \to 18:30}$

(3-XI-20)

$$\left.\begin{aligned}
\mathbb{H}\otimes\mathbb{C} &\longrightarrow M_2(\mathbb{C})\\
\tfrac{1}{2}(1+i\otimes i) &\longmapsto \begin{pmatrix}1&0\\0&0\end{pmatrix}\\
\tfrac{1}{2}(1-i\otimes i) &\longmapsto \begin{pmatrix}0&0\\0&1\end{pmatrix}\\
? &\longmapsto \begin{pmatrix}0&1\\0&0\end{pmatrix}\\
? &\longmapsto \begin{pmatrix}0&0\\1&0\end{pmatrix}
\end{aligned}\right\}$$