On Summing Operators on JB*-triples

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Abstract

In this paper we introduce 2-JB*-triple-summing operators on real and complex JB*-triples. These operators generalize 2-C*-summing operators on C*-algebras. We also obtain a Pietsch's factorization theorem in the setting of 2-JB*-triple-summing operators on JB*-triples.

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1 Introduction.

Let X, Y be Banach spaces, $0 < p < \infty$, and $T : X \to Y$ a bounded linear operator. We say that T is p-summing if there is a constant $C \geq 0$ such that for any finite sequence $(x_1, \ldots, x_n)$ of X we have

$$\left(\sum_{k=1}^{n} \|T(x_k)\|^p\right)^{\frac{1}{p}} \leq C \sup\left\{ \left(\sum_{k=1}^{n} |f(x_k)|^p\right)^{\frac{1}{p}} : f \in X^*, \ \|f\| \leq 1 \right\}.$$

In 1978, G. Pisier [20] introduced the following extension of the p-summing operators in the setting of C*-algebras. Let T be a bounded linear operator from a C*-algebra $\mathcal{A}$ to a Banach space Y, and $0 < p < \infty$. We say that T

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is p-C*-summing if there exists a positive constant C such that for any finite sequence $(a_1, \dots, a_n)$ of hermitian elements of $\mathcal{A}$ we have

$$\left(\sum_{i=1}^{n} \|T(a_i)\|^p\right)^{\frac{1}{p}} \leq C \left\| \left(\sum_{i=1}^{n} |a_i|^p\right)^{\frac{1}{p}} \right\|, \tag{1}$$

where, for x in $\mathcal{A}$, the "modulus" is defined by $|x|^2 := \frac{1}{2}(xx^* + x^*x)$. The smallest constant C for which (1) holds is denoted $C_p(T)$. It is well known that every p-summing operator from a C*-algebra to a Banach space is p-C*-summing but the converse is false in general (compare [20, Remark 1.2]).

In [20] G. Pisier proved a Pietsch's factorization theorem for p-C*-summing operators. Indeed, if $T : \mathcal{A} \to Y$ is a p-C*-summing operator from a C*-algebra to a complex Banach space then there is a norm-one positive linear functional φ in $\mathcal{A}^*$ such that

$$\|T(x)\| \leq C_p(T) \ \ (\varphi(|x|^p))^{\frac{1}{p}}$$

for every hermitian element x in $\mathcal{A}$.

Complex JB*-triples were introduced by W. Kaup in the study of Bounded Symmetric Domains in complex Banach spaces ([15], [14]). The class of complex JB*-triples includes all C*-algebras and all JB*-algebras.

The aim of this paper is the study of summing operators on real and complex JB*-triples. In Section 2 we introduce the natural definition of p-JB*-summing operators in the setting of JB*-algebras. We obtain a Pietsch's factorization theorem for p-JB*-summing operators. Section 3 deals with the definition and study of 2-JB*-triple-summing operators in the setting of complex JB*-triples. Operators which generalize 2-C*-summing and 2-JB*-summing operators on C*-algebras and JB*-algebras, respectively. For the most general class of 2-JB*-triple-summing operators, we obtain a Pietsch's factorization theorem in the setting of JB*-triples (Theorems 3.5 and 3.6). It is worth mentioning that in the proof of this result, the so-called "Little Grothendieck's inequality" for JB*-triples [18] play a very important role. In the last section we establish analogous results in the setting of real JB*-triples. We also discuss the relations between 2-summing and 2-JB*-triple-summing operators.

Let X be a Banach space. Through the paper we denote by B_X, S_X, and X^* the closed unit ball, the unit sphere, and the dual space, respectively, of X. I_X will denote the identity operator on X, J_X the natural embedding of X in its bidual X^{**}, and if Y is another Banach space, then $BL(X,Y)$ stands for the Banach space of all bounded linear operators from X to Y. We usually write $BL(X)$ instead of $BL(X,X)$.

2 Summing Operators on JB*-algebras

Let $\mathcal{A}$ be a JB*-algebra. Given $x \in \mathcal{A}$, the modulus $|x|$ is defined by $|x|^2 := x \circ x^*$ for all $x \in \mathcal{A}$. Given a norm-one positive linear functional $\psi \in \mathcal{A}^*$, the mapping $(x,y) \mapsto (x/y)_\psi := \psi(x \circ y^*)$ is a positive sesquilinear form on $\mathcal{A}$. If we denote $N_\psi := \{x \in \mathcal{A} : \psi(x \circ x^*) = 0\}$, then the quotient $\mathcal{A}/N_\psi$ can be completed to a Hilbert space, which is denoted by H_ψ. The natural quotient map of $\mathcal{A}$ on H_ψ is denoted by J_ψ. Inspired by the definition of p-C*-summing operators, we introduced the following concept of p-JB*-summing operator in the setting of JB*-algebras.

Definition 2.1. *Let $0 < p < \infty$. A bounded linear operator T from a JB*-algebra $\mathcal{A}$ to a Banach space Y is said to be p-JB*-summing if there exists a positive constant C such that for any finite sequence $(a_1, \dots, a_n)$ of hermitian elements of $\mathcal{A}$ we have*

$$\left(\sum_{i=1}^{n} \|T(a_i)\|^p \right)^{\frac{1}{p}} \leq C \left\| \left(\sum_{i=1}^{n} |a_i|^p \right)^{\frac{1}{p}} \right\|. \tag{2}$$

The smallest constant C for which (2) holds is denoted $C_p(T)$.

Remark 2.2. *Let $\mathcal{A}$ be a C*-algebra. Then $\mathcal{A}$ is a JB*-algebra with respect $x \circ y := \frac{1}{2}(xy + yx)$. In this case, it is easy to see that a bounded linear operator from $\mathcal{A}$ to a Banach space is p-C*-summing if and only if it is p-JB*-summing.*

The next result is an extension of Pietsch's factorization theorem ([9]) in the JB*-algebra setting which is a verbatim extension of Pisier's analogous result for C*-algebras [20, Proposition 1.1].

Proposition 2.3. *Let $\mathcal{A}$ be a JB*-algebra, Y a Banach space, and $T : \mathcal{A} \to Y$ a p-JB*-summing operator. Then there exists a norm-one positive linear functional φ on $\mathcal{A}$ such that*

$$\|T(x)\| \leq C_p(T) \ \ (\varphi(|x|^p))^{\frac{1}{p}}$$

for every hermitian element x in $\mathcal{A}$.

Proof. Let K denote the set of all positive linear functionals on $\mathcal{A}$ with norm less or equal to 1. Then K is a convex $\sigma(\mathcal{A}^*, \mathcal{A})$-compact subset of $\mathcal{A}^*$ and

$$\|a\| = \sup_{f \in K} |f(a)| \tag{3}$$

for every hermitian element $a \in \mathcal{A}$ [12].

Let us denote by $\mathcal{C}$ the set of all continuous functions on K of the form

$$F_{\{a_1,\dots,a_n\}}(f) := C_p(T)^p \ f(\sum_{i=1}^{n} |a_i|^p) - \sum_{i=1}^{n} \|Ta_i\|^p,$$

where $(a_1, \dots, a_n)$ is a finite collection of hermitian elements in $\mathcal{A}$. Then $\mathcal{C}$ is a convex cone in $C(K)$. Moreover, since T is p-JB*-summing, (3) assures that $\mathcal{C}$ is disjoint from the open cone $\mathcal{O} := \{\Phi \in C(K) : \max \Phi < 0\}$. By the Hahn-Banach theorem there exists a positive measure λ on K such that

$$\int_K F_{\{a_1,\dots,a_n\}}(k)\lambda(dk) \geq 0$$

for every finite collection of hermitian elements $(a_1, \dots, a_n) \in \mathcal{A}$. We can suppose that λ is a probability measure on K. Finally taking $\varphi(x) := \int_K k(x)\lambda(dk) \ \ (x \in \mathcal{A})$, we finish the proof. □

Remark 2.4. *Let $\mathcal{A}$, Y, and T be as in Proposition 2.3 above with $p = 2$. If $x \in \mathcal{A}$, then $x = a + ib$ with $a^* = a$, $b^* = b$, and hence $|x|^2 = a^2 + b^2$. Therefore*

$$\|T(x)\| \leq \sqrt{2} C_2(T)(\varphi(|x|^2))^{\frac{1}{2}}.$$

The next result is a weak* version of Pietsch factorization theorem for JB*-algebras (Proposition 2.3) and a extension of [21, Lemma 4.1] in the JBW*-algebra setting. We recall that a JBW*-algebra is a JB*-algebra which is also a dual Banach space [12].

Proposition 2.5. *Let $\mathcal{A}$ be a JBW*-algebra, Y a Banach space, and $T : \mathcal{A} \to Y^*$ a p-JB*-summing operator which is also weak*- continuous. Then there exists a norm-one positive linear functional φ in $\mathcal{A}_*$ such that*

$$\|T(x)\| \leq C_p(T) \ \ (\varphi(|x|^p))^{\frac{1}{p}}$$

for every hermitian element x in $\mathcal{A}$.

Proof. By [12, 4.4.17] there exists a central projection $e \in \mathcal{A}^{**}$ such that

$$L_e : \mathcal{A} \to e \circ \mathcal{A}^{**}$$

$$L_e(x) := e \circ x$$

is an isomorphism and $\mathcal{A}_* = L_e^*(\mathcal{A}^*)$. By Proposition 2.3, there exists a norm-one positive linear functional $\psi \in \mathcal{A}^*$ such that

$$\|T(x)\| \leq C_p(T) \ \ (\psi(|x|^p))^{\frac{1}{p}}$$

for every hermitian element x in $\mathcal{A}$. Now we take $\varphi := L_e^*(\psi) \in \mathcal{A}_*$, $f \in S_Y$ and compute

$$< f, T(x) >=< T^*(f), x >=< L_e T^*(f), x >\leq \|T^{**}(e \circ x)\|$$

$$\leq C_p(T) \ \ (\psi(|e \circ x|^p))^{\frac{1}{p}} = C_p(T) \ \ (\psi(e \circ |x|^p))^{\frac{1}{p}} = C_p(T) \ \ (\varphi(|x|^p))^{\frac{1}{p}} .$$

Finally taking supremum over $f \in S_Y$ we finish the proof. □

3 Summing Operators on JB*-triples

We recall that a (complex) JB*-triple is a complex Banach space $\mathcal{E}$ with a continuous triple product $\{.,.,.\} : \mathcal{E} \times \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ which is bilinear and symmetric in the outer variables and conjugate linear in the middle variable, and satisfies:

1. (Jordan Identity) $L(a,b)\{x,y,z\} = \{L(a,b)x,y,z\} - \{x,L(b,a)y,z\} + \{x,y,L(a,b)z\}$ for all a,b,c,x,y,z in $\mathcal{E}$, where $L(a,b)x := \{a,b,x\}$;

2. The map $L(a,a)$ from $\mathcal{E}$ to $\mathcal{E}$ is an hermitian operator with non negative spectrum for all a in $\mathcal{E}$;

3. $\|\{a,a,a\}\| = \|a\|^3$ for all a in $\mathcal{E}$.

We recall that a bounded linear operator on a complex Banach space is said to be *hermitian* if $\|\exp(i\lambda T)\| = 1$ for all $\lambda \in \mathbb{R}$.

It is worth mentioning that every C*-algebra is a (complex) JB*-triple with respect to $\{a,b,c\} = \frac{1}{2}(ab^*c + cb^*a)$ and also every JB*-algebra with respect to $\{a,b,c\} = (a \circ b^*) \circ c + (c \circ b^*) \circ a - (a \circ c) \circ b^*$. We refer to [23],

[24] and [8] for recent surveys on the theory of JB*-triples. A JBW*-triple is a JB*-triple which is a dual Banach space. If $\mathcal{E}$ is a JB*-triple then $\mathcal{E}^{**}$ is a JBW*-triple [11]. It is well known that every JBW*-triple has a unique predual and the triple product is separately weak*-continuous [3].

Let $\mathcal{E}$ be a JB*-triple and φ a norm-one functional in $\mathcal{E}^*$. By [1, Proposition 1.2] the map $(x, y) \mapsto \varphi\{x, y, z\}$ is a positive sequilinear form on $\mathcal{E}$, where z is any norm-one element of $\mathcal{E}^{**}$ verifying $\varphi(z) = 1$ (If $\mathcal{E}$ is a JBW*-triple and $\varphi \in S_{\mathcal{E}_*}$, then z can be chosen in $S_{\mathcal{E}}$).

If we define $N_\varphi := \{x \in \mathcal{E} : \|x\|_\varphi = 0\}$, the completion H_φ of $\mathcal{E}/N_\varphi$ is a Hilbert space with respect to the norm $\|.\|_\varphi$. Throughout the paper the natural quotient map of $\mathcal{E}$ on H_φ will be denoted by J_φ.

Let $\mathcal{W}$ be a JBW*-triple. The *strong** topology of $\mathcal{W}$, denoted by $S^*(\mathcal{W}, \mathcal{W}_*)$, is the topology on $\mathcal{W}$ generated by the family $\{\|.\|_\varphi : \varphi \in S_{\mathcal{W}_*}\}$.

The following definition is the natural extension of the 2-summing operators in the setting of JB*-triples.

Definition 3.1. *Let $\mathcal{E}$ be a JB*-triple and Y a Banach space. An operator $T : \mathcal{E} \to Y$ is said to be 2-JB*-triple-summing if there exists a positive constant C such that for every finite sequence $(x_1, \ldots, x_n)$ of elements in $\mathcal{E}$ we have*

$$\sum_{i=1}^{n} \|T(x_i)\|^2 \leq C \left\| \sum_{i=1}^{n} L(x_i, x_i) \right\|. \tag{4}$$

The smallest constant C for which (4) *holds is denoted $C_2(T)$.*

Let X be a Banach space, and u a norm-one element in X. The set of states of X relative to u, $D(X, u)$, is defined as the non empty, convex, and weak*-compact subset of X^* given by

$$D(X, u) := \{\Phi \in B_{X^*} : \Phi(u) = 1\}.$$

For $x \in X$, the *numerical range* of x relative to u, $V(X, u, x)$, is given by $V(X, u, x) := \{\Phi(x) : \Phi \in D(X, u)\}$. The *numerical radius* of x relative to u, $v(X, u, x)$, is given by

$$v(X, u, x) := \max\{|\lambda| : \lambda \in V(X, u, x)\}.$$

It is well known that a bounded linear operator T on a complex Banach space X is hermitian if and only if $V(BL(X), I_X, T) \subseteq \mathbb{R}$ (compare [5, Corollary 10.13]). If T is a bounded linear operator on X, then we have $V(BL(X), I_X, T) = \overline{co}\, W(T)$ where

$$W(T) = \{x^*(T(x)) : (x, x^*) \in \Gamma\},$$

and $\Gamma \subseteq \{(x, x^*) : x \in S_X, x^* \in S_{X^*}, x^*(x) = 1\}$ verifies that its projection onto the first coordinate is norm dense in S_X [4, Theorem 9.3]. Moreover, the numerical radius of T can be calculated as follows

$$v(BL(X), I_X, T) = \sup\{|x^*(T(x))| : (x, x^*) \in \Gamma\}.$$

In particular if $X = Y^*$, then by the Bishop-Phelps-Bollobás theorem, it follows that

$$v(BL(X), I_X, T) = \sup\{|x^*(T(x))| : x \in S_X, x^* \in S_Y, x^*(x) = 1\}.$$

Remark 3.2. *Let $\mathcal{A}$ be a JB*-algebra with unit 1, Y a Banach space, and $T : \mathcal{A} \to Y$ a 2-JB*-triple-summing operator (regarded $\mathcal{A}$ as a JB*-triple). We have*

$$\sum_{i=1}^{n} \|T(x_i)\|^2 \leq C_2(T) \left\| \sum_{i=1}^{n} L(x_i, x_i) \right\| \tag{5}$$

for every finite sequence $(x_1, \ldots, x_n)$ of elements in $\mathcal{A}$. Since $S := \sum_{i=1}^{n} L(x_i, x_i)$ is an hermitian operator on $\mathcal{A}$, Sinclair's theorem [5, Theorem 11.17] assures that

$$\|S\| = \sup\{|\Phi(S(z))| : z \in S_{\mathcal{A}}, \Phi \in S_{\mathcal{A}^*}, \Phi(z) = 1\}.$$

It is worth mentioning that $\Phi(S(z)) \geq 0$ for such Φ and z. Let $z \in S_{\mathcal{A}}$ and $\Phi \in S_{\mathcal{A}^*}$ with $\Phi(z) = 1$. Let us define $\Psi(x) := \Phi(x \circ z)$, then we have $\Psi \in S_{\mathcal{A}^*}$, $\Psi(1) = \Phi(z) = 1$, and

$$\Psi(L(x, x)(1)) = \Phi(L(x, x)(1) \circ z)$$

$$= \frac{1}{2}\Phi(\{x, x, z\} + \{x^*, x^*, z\}) = \frac{1}{2}(\|x\|_\Phi + \|x^*\|_\Phi)$$

$$\geq \frac{1}{2}\|x\|_\Phi = \frac{1}{2}\Phi(L(x, x)(z))$$

for all $x \in \mathcal{A}$. Therefore $\Phi(S(z)) \leq 2\Psi(S(1))$ and hence

$$\|S\| \leq 2\ \sup\{\Psi(S(z)) : \Psi \in S_{\mathcal{A}^*}, \Phi(1) = 1\}$$

$$= 2\ \sup\{\Psi(\sum_{i=1}^{n}|x_i|^2) : \Psi \in S_{\mathcal{A}^*}, \Phi(1) = 1\} = 2\Big\|\sum_{i=1}^{n}|x_i|^2\Big\|.$$

It follows from (5) *that T is 2-JB*-summing. This shows that every 2-JB*-triple-summing operator from a unital JB*-algebra to a Banach space is 2-JB*-summing. Conversely, the inequality*

$$\Big\|\sum_{i=1}^{n}|x_i|^2\Big\| = \Big\|\sum_{i=1}^{n}L(x_i,x_i)(1)\Big\| \leq \Big\|\sum_{i=1}^{n}L(x_i,x_i)\Big\|,$$

shows that every 2-JB-summing operator from a unital JB*-algebra is also 2-JB*-triple-summing.*

In 1987 T. Barton and Y. Friedman [1, Corollary 3.1] established a Ringrose-type inequality for JB*-triples. However the Barton-Friedman proof of this inequality is based in [1, Theorem 1.3], result which has a gap (see [17] and [18]). Now we can follow the same ideas to prove this Ringrose-type inequality, but replacing [1, Theorem 1.3] by [18, Theorem 3]. Given a JB*-triple $\mathcal{E}$ and norm-one elements $\varphi_1, \varphi_2 \in \mathcal{E}^*$ we denote by $\|.\|_{\varphi_1,\varphi_2}$ the prehilbert seminorm on $\mathcal{E}$ given by $\|x\|^2_{\varphi_1,\varphi_2} := \|x\|^2_{\varphi_1} + \|x\|^2_{\varphi_2}$.

Proposition 3.3. *Let $\mathcal{E}$ and $\mathcal{F}$ be JB*-triples, $T : \mathcal{E} \to \mathcal{F}$ a bounded linear operator, and $x_1, \ldots, x_n$ in $\mathcal{E}$. Then*

$$\Big\|\sum_{i=1}^{n}L(T(x_i),T(x_i))\Big\| \leq 2\|T\|^2\Big\|\sum_{i=1}^{n}L(x_i,x_i)\Big\|.$$

Proof. Let us denote $S := \sum_{i=1}^{n} L(T(x_i),T(x_i))$. Then S is an hermitian operator on $\mathcal{F}$. By Sinclair's theorem [5, Theorem 11.17]

$$\|S\| = \sup\{|\psi(S(z))| : z \in S_{\mathcal{F}}, \psi \in S_{\mathcal{F}^*}, \psi(z) = 1\}.$$

Note that $\psi(S(Z)) > 0$ for every such ψ and z. Fix $\varepsilon > 0$ and choose ψ and z such that

$$\|S\| \leq \psi(S(z)) + \varepsilon.$$

The mapping $J_\psi T : \mathcal{E} \to H_\psi$ is a bounded linear operator. Let $m \in \mathbb{N}$. By [18, Theorem 3], there are norm-one functionals $\varphi_{1,m}, \varphi_{2,m} \in \mathcal{E}^*$ such that

$$\|J_\psi T(x)\| \leq (\sqrt{2} + \frac{1}{m})\|T\|\ (\|x\|^2_{\varphi_{1,m}} + \frac{1}{m}\|x\|^2_{\varphi_{2,m}})^{\frac{1}{2}},\ \text{i. e.}$$

$$\psi\{T(x),T(x),z\} \leq (\sqrt{2}+\frac{1}{m})^2\|T\|^2\ (\varphi_{1,m}\{x,x,e_{1,m}\}+\frac{1}{m}\varphi_{2,m}\{x,x,e_{2,m}\})$$

for all $x \in \mathcal{E}$, where $e_{1,m}, e_{2,m} \in S_{\mathcal{E}^{**}}$ verify $\varphi_{i,m}(e_{i,m}) = 1$ for $i \in \{1,2\}$.

Therefore

$$\|S\| - \varepsilon \leq \psi(S(z)) \leq (\sqrt{2}+\frac{1}{m})^2\|T\|^2\ (\varphi_{1,m}(\sum_{i=1}^{n} L(x_i,x_i)e_{1,m})+$$

$$\frac{1}{m}\varphi_{2,m}(\sum_{i=1}^{n} L(x_i,x_i)e_{2,m})) \leq (\sqrt{2}+\frac{1}{m})^2(1+\frac{1}{m})\|T\|^2\ \Big\|\sum_{i=1}^{n} L(x_i,x_i)\Big\|.$$

Finally, letting $\varepsilon \to 0, m \to \infty$, we get

$$\Big\|\sum_{i=1}^{n} L(T(x_i),T(x_i))\Big\| \leq 2\|T\|^2\Big\|\sum_{i=1}^{n} L(x_i,x_i)\Big\|.$$

□

From the above proposition, we immediately obtain the following corollary.

Corollary 3.4. *Let $\mathcal{E}$ and $\mathcal{F}$ be JB*-triples, Y a Banach space, $T:\mathcal{F}\to Y$ a 2-JB*-triple-summing operator, and $R:\mathcal{E}\to\mathcal{F}$ a bounded linear operator. Then $TR:\mathcal{E}\to Y$ is a 2-JB*-triple-summing operator.*

Now we deal with the following characterization of 2-JB*-triple-summing operators from a JBW*-triple to a complex Banach space which generalizes Pietsch's factorization theorem for C*-algebras [21, Theorem 3.2].

Theorem 3.5. *Let T be a weak*-continuous linear operator from a JBW*-triple $\mathcal{W}$ with values in a Banach space Y^*. The following assertions are equivalent.*

1. *T is 2-JB*-triple-summing.*

2. *There are norm-one functionals φ_1,φ_2 in $\mathcal{W}_*$ and a positive constant $C(T)$ such that*

$$\|T(x)\| \leq C(T)\|x\|_{\varphi_1,\varphi_2}$$

   for all $x \in \mathcal{W}$.

Proof. 1 $\Rightarrow$ 2.– By [6, Proposition 2] there is a (unital) JBW*-algebra $\mathcal{A}$ and a contractive projection $P : \mathcal{A} \to \mathcal{W}$. Actually, by [23, Theorem D.20], P can be supposed weak*-continuous. Since, by Corollary 3.4 and Remark 3.2, it follows that $T\ P : \mathcal{A} \to Y^*$ is a 2-JB*-summing operator, which is also weak*-continuous, we conclude, by Theorem 2.5, that there exists a norm-one positive functional $\psi \in \mathcal{A}_*$ such that

$$\|TP(\alpha)\| \leq \sqrt{2\ C_2(T)}(\psi(\alpha \circ \alpha^*))^{\frac{1}{2}}$$

for all $\alpha \in \mathcal{A}$.

It is worth mentioning that, by the same arguments given in the proof of [22, Corollary 1], the natural quotient map J_ψ is weak*-continuous. Therefore, we can apply [18, Theorem 3] to the restriction $J_\psi|_{\mathcal{W}} : \mathcal{W} \to H_\psi$ to get norm-one functionals $\varphi_1, \varphi_2 \in \mathcal{W}_*$ such that

$$\psi(x \circ x^*) \leq 4\ \|x\|^2_{\varphi_1,\varphi_2},$$

and hence

$$\|T(x)\| \leq 2\sqrt{2C_2(T)}\ \|x\|_{\varphi_1,\varphi_2}$$

for all $x \in \mathcal{W}$.

2 $\Rightarrow$ 1.– Let $(x_1, \ldots, x_n)$ be a finite sequence of elements of $\mathcal{W}$, and $e_1, e_2 \in S_{\mathcal{W}}$ such that $\varphi_i(e_i) = 1$ for $i \in \{1, 2\}$. Then we have

$$\sum_{i=1}^{n} \|T(x_i)\|^2 \leq C(T)^2\ \sum_{i=1}^{n} \|x_i\|^2_{\varphi_1,\varphi_2}$$

$$= C(T)^2\ (\varphi_1(\sum_{i=1}^{n} L(x_i, x_i)(e_1)) + \varphi_2(\sum_{i=1}^{n} L(x_i, x_i)(e_2)))$$

$$\leq 2C(T)^2 \Big\| \sum_{i=1}^{n} L(x_i, x_i) \Big\|.$$

Inequality which shows that T is 2-JB*-triple-summing. □

Let $\mathcal{E}$ be a complex JB*-triple and $\Phi \in D(BL(\mathcal{E}), I_{\mathcal{E}})$. Since for every $x \in \mathcal{E}$, the operator $L(x, x)$ is hermitian and has non-negative spectrum, it follows from [5, Lemma 38.3] that the mapping $(x, y) \to \Phi(L(x, y))$ from $\mathcal{E} \times \mathcal{E}$ to $\mathbb{C}$ becomes a positive sesquilinear form on $\mathcal{E}$. Then we define the prehilbert seminorm $|||.|||_\Phi$ on $\mathcal{E}$ by $|||x|||^2_\Phi := \Phi(L(x, x))$.

Our next result is the natural extension of Pietsch's factorization theorem in the setting of JB*-triples.

Theorem 3.6. *Let T be a bounded operator from a JB*-triple $\mathcal{E}$ with values in a Banach space Y. The following assertions are equivalent.*

1. *T is 2-JB*-triple-summing.*

2. *There is a state $\Psi \in D(BL(\mathcal{E}), I_{\mathcal{E}})$ and a positive constant $C(T)$ such that*
$$\|T(x)\| \leq C(T)\||x\||_{\Psi}$$
for every $x \in \mathcal{E}$.

3. *There are norm-one functionals φ_1, φ_2 in $\mathcal{E}^*$ and a positive constant $C(T)'$ such that*
$$\|T(x)\| \leq C(T)'\|x\|_{\varphi_1,\varphi_2}$$
for all $x \in \mathcal{E}$.

Proof. $1 \Rightarrow 2$.– Let K denote the set of states of $BL(\mathcal{E})$ relative to $I_{\mathcal{E}}$. Then K is a non empty, convex, and weak*-compact subset of $BL(\mathcal{E})^*$. Moreover, by Sinclair's theorem [5, Theorem 11.17],

$$\|T\| = \sup_{\Phi \in K} |\Phi(T)| \tag{6}$$

for every hermitian operator T on X.

Let us denote by $\mathcal{C}$ the set of all continuous functions on K of the form

$$F_{\{x_1,\dots,x_n\}}(\Phi) := C_2(T)\Phi(\sum_{i=1}^{n} L(x_i, x_i)) - \sum_{i=1}^{n} \|T(x_i)\|^2$$

where $n \in \mathbb{N}$ and $\{x_1, \dots, x_n\} \subset \mathcal{E}$. Since for every $\{x_1, \dots, x_n\} \subset \mathcal{E}$, the map $\sum_{i=1}^{n} L(x_i, x_i)$ is an hermitian operator on $\mathcal{E}$ and T is 2-JB*-triple-summing, (6) assures that $\mathcal{C}$ is disjoint from the open cone $\mathcal{O} := \{\varphi \in C(K) : \max \varphi < 0\}$. Therefore, by the Hahn-Banach theorem there is a probability measure μ on K such that

$$\int_K F_{\{x_1,\dots,x_n\}}(k)\ \mu(dk) \geq 0$$

for every finite collection of elements $\{x_1, \dots, x_n\} \in \mathcal{E}$. Finally taking $\Psi(T) := \int_K T(k)\ \mu(dk)$ we obtain 2.

$2 \Rightarrow 3.-$ Let Ψ the state given in 2. The map $|||.|||_\Psi$ is a pre-Hilbert seminorm on $\mathcal{E}$. Denoting $N := \{x \in \mathcal{E} : |||x|||_\Psi = 0\}$, then the quotient $\mathcal{E}/N$ can be completed to a Hilbert space H. Let us denote by Q the natural quotient map from $\mathcal{E}$ to H. By [18, Corollary 1] (see also [19, Corollary 1.11]) there are norm-one functionals $\varphi_1, \varphi_2 \in \mathcal{E}^*$ such that the inequality

$$\|Q(x)\| = |||x|||_\Psi \leq 2\|x\|_{\varphi_1,\varphi_2}$$

holds for every $x \in \mathcal{E}$. Then it follows that

$$\|T(x)\| \leq 2C(T)\|x\|_{\varphi_1,\varphi_2}$$

for every $x \in \mathcal{E}$.

The implication $3 \Rightarrow 1.-$ follows as $(2 \Rightarrow 1)$ in Theorem 3.5. □

Let $T : \mathcal{E} \to Y$ be a 2-JB*-triple-summing operator from a JB*-triple to a Banach space. By Theorem 3.6 above, there are norm-one functionals φ_1, φ_2 in $\mathcal{E}^*$ and a positive constant $C(T)'$ such that

$$\|T(x)\| \leq C(T)'\|x\|_{\varphi_1,\varphi_2} \tag{7}$$

for all $x \in \mathcal{E}$. Let $\alpha \in \mathcal{E}^{**}$. Since by [2, Theorem 3.2], the strong*-topology of $\mathcal{E}^{**}$ is compatible with the duality, it follows that there is a net $(x_\lambda) \subseteq \mathcal{E}$ converging to α in the strong*-topology and hence also in the weak*-topology of $\mathcal{E}^{**}$. Since the seminorm $\|.\|_{\varphi_1,\varphi_2}$ is strong*-continuous, by (7) and the weak*-lower semicontinuity of the norm we have

$$\|T^{**}(\alpha)\| \leq C(T)'\|\alpha\|_{\varphi_1,\varphi_2}.$$

Therefore, by Theorem 3.6 we conclude that T^{**} is 2-JB*-triple-summing. We have thus proved the following lemma.

Lemma 3.7. *Let $T : \mathcal{E} \to Y$ be a 2-JB*-triple-summing operator from a JB*-triple to a Banach space. Then there are norm-one functionals φ_1, φ_2 in $\mathcal{E}^*$ and a positive constant $C(T)'$ such that*

$$\|T^{**}(\alpha)\| \leq C(T)'\|\alpha\|_{\varphi_1,\varphi_2}$$

for all $\alpha \in \mathcal{E}^{**}$. In particular T^{**} is 2-JB-triple-summing.*

Remark 3.8. *Let $\mathcal{A}$ be a JB*-algebra. By [12, Proposition 3.5.4] $\mathcal{A}$ has an increasing approximate identity of hermitian elements, i. e., there is a net $(u_\lambda)_\Lambda \subseteq \mathcal{A}$ where Λ is a directed set, $u_\lambda^* = u_\lambda$, $\|u_\lambda\| \leq 1$, and $\|u_\lambda \circ x - x\| \to 0$ for every $x \in \mathcal{A}$. Then*

$$\|L(x,x)(u_\lambda) - |x|^2\| = \||x|^2 \circ u_\lambda + (u_\lambda \circ x^*) \circ x - (u_\lambda \circ x) \circ x^* - |x|^2\| \to 0$$

and hence

$$\|\sum_{i=1}^{n} |x_i|^2\| = \lim_\lambda \|\sum_{i=1}^{n} L(x_i,x_i)(u_\lambda)\| \leq \|\sum_{i=1}^{n} L(x_i,x_i)\|$$

for every finite sequence $(x_1, \ldots, x_n) \in \mathcal{A}$. It follows that every 2-JB-summing operator from $\mathcal{A}$ to a Banach space is 2-JB*-triple-summing (regarded $\mathcal{A}$ as a JB*-triple). Conversely if $T : \mathcal{A} \to Y$ is a 2-JB*-triple-summing operator then, by Lemma 3.7, $T^{**} : \mathcal{A}^{**} \to Y^{**}$ is a 2-JB*-triple-summing operator. Since $\mathcal{A}^{**}$ is a unital JBW*-algebra, it follows, by Remark 3.2, that T^{**} (and hence T) is a 2-JB*-summing operator.*

4 Summing Operators on real JB*-triples

Real JB*-triples were defined by J. M. Isidro, W. Kaup, and A. Rodríguez [13], as norm-closed real subtriples of complex JB*-triples. In [13], it is shown that given a real JB*-triple E, then there exists a unique complex JB*-triple structure on its complexification $\widehat{E} = E \oplus iE$ and a unique conjugation (conjugate-linear isometry of period 2) τ on $\widehat{E}$ such that $E = \widehat{E}^\tau := \{z \in \widehat{E} : \tau(z) = z\}$. All JB-algebras, all real C*-algebras and obviously all complex JB*-triples are examples of real JB*-triples. By a real JBW*-triple we mean a real JB*-triple whose underlying Banach space is a dual Banach space. As in the complex case, the triple product of every real JBW*-triple is separately weak*-continuous [16], and the bidual $\mathcal{E}^{**}$ of a real JB*-triple $\mathcal{E}$ is a real JBW*-triple whose triple product extends the one of $\mathcal{E}$ [13]. Noticing that every real JBW*-triple is a real form of a complex JBW*-triple [13], it follows easily that, if W is a real JBW*-triple and if φ is a norm-one element in W_*, then, for $z \in W$ such that $\varphi(z) = \|z\| = 1$, the mapping $x \mapsto (\varphi\{x,x,z\})^{\frac{1}{2}}$ is a prehilbert seminorm on W (not depending on z). Such a seminorm will be denoted by $\|.\|_\varphi$. The *strong** topology on W, denoted by $S^*(W, W_*)$, is the topology on W generated by the family $\{\|.\|_\varphi : \varphi \in S_{W_*}\}$.

As in the complex case, we say that a linear operator T from a real JB*-triple E to a real Banach space Y is 2-JB*-triple-summing if there exists a positive constant C such that for every finite sequence $(x_1, \dots, x_n)$ of elements in E we have

$$\sum_{i=1}^{n} \|T(x_i)\|^2 \leq C \left\| \sum_{i=1}^{n} L(x_i, x_i) \right\|. \tag{8}$$

The smallest constant C for which (8) holds is again denoted $C_2(T)$.

Let $T : E \to F$ be a bounded linear operator between real JB*-triples and let $M > \sqrt{2}$. Let us consider $\widehat{T} : \widehat{E} \to \widehat{F}$ the natural complex linear extension of T. By Proposition 3.3

$$\left\| \sum_{k=1}^{n} L(\widehat{T}(z_k), \widehat{T}(z_k)) \right\| \leq 2M^2 \|\widehat{T}\|^2 \left\| \sum_{k=1}^{n} L(z_k, z_k) \right\|,$$

for every finite sequence $(z_1, \dots, z_n) \subseteq \widehat{E}$. In particular, the inequality

$$\left\| \sum_{k=1}^{n} L(T(x_k), T(x_k)) \right\| \leq 8M^2 \|T\|^2 \left\| \sum_{k=1}^{n} L(x_k, x_k) \right\|,$$

holds for every finite sequence $(x_1, \dots, x_n) \subseteq E$. We deduce, as in the complex case, the following result.

Corollary 4.1. *Let E and F be real JB*-triples, Y a real Banach space, $T : F \to Y$ a 2-JB*-triple-summing operator, and $R : E \to F$ a bounded linear operator. Then $TR : E \to Y$ is a 2-JB*-triple-summing operator.*

Remark 4.2. *Let E be a real JB*-triple, Y a real Banach space, and $T : E \to Y$ a 2-JB*-triple-summing operator. We denote by $\tilde{Y}$ the complex Banach space $Y \oplus iY$ equipped with the norm*

$$\|x + iy\|_c := \sup\{\|\alpha x - \beta y\| : \alpha + i\beta \in \mathbb{C} \text{ with } |\alpha + i\beta| = 1\}.$$

Then T can be extended to a complex linear operator $\widehat{T} : \widehat{E} \to \tilde{Y}$. We claim that $\widehat{T}$ is 2-JB-triple-summing. Indeed, given $(x_1 + iy_1, \dots, x_n + iy_n) \subseteq \widehat{E}$ we have*

$$\begin{aligned} \sum_{k=1}^{n} \|\widehat{T}(x_k + iy_k)\|^2 &\leq 2 \sum_{k=1}^{n} \|T(x_k)\|^2 + \|T(y_k)\|^2 \qquad (9) \\ &\leq C_2(T) \left\| 2 \sum_{k=1}^{n} L(x_k, x_k) + L(y_k, y_k) \right\| \end{aligned}$$

Now, since $S := \sum_{k=1}^n L(x_k, x_k) + L(y_k, y_k)$ *is an hermitian operator on* $\widehat{E}$ *it follows, by Sinclair's theorem, that*

$$\|2S\| = \sup\{2\Phi(S(z)) : \Phi \in S_{\widehat{E}^*}, z \in S_{\widehat{E}}, \Phi(z) = 1\}$$

$$= \sup\{2\sum_{k=1}^n \|x_k\|_\Phi^2 + \|y_k\|_\Phi^2 : \Phi \in S_{\widehat{E}^*}, z \in S_{\widehat{E}}, \Phi(z) = 1\}$$

$$= \sup\{\sum_{k=1}^n \|x_k + iy_k\|_\Phi^2 + \|\tau(x_k + iy_k)\|_\Phi^2 : \Phi \in S_{\widehat{E}^*}, z \in S_{\widehat{E}}, \Phi(z) = 1\}$$

$$\leq \Big\| \sum_{k=1}^n L(x_k + iy_k, x_k + iy_k)\Big\| + \Big\| \sum_{k=1}^n L(\tau(x_k + iy_k), \tau(x_k + iy_k))\Big\|$$

$$= 2\Big\| \sum_{k=1}^n L(x_k + iy_k, x_k + iy_k)\Big\|.$$

Therefore, we conclude by (9) *that* $\widehat{T}$ *is 2-JB*-triple-summing and* $C_2(\widehat{T}) \leq 2C_2(T)$.

Let E be a real JB*-triple. Following [19], we known that given $\Phi \in D(BL(E), I_E)$ then the mapping $(x, y) \to \Phi(L(x, y))$ from $E \times E$ to $\mathbb{R}$ is a positive symmetric bilinear form on E, and hence $|||x|||_\Phi^2 := \Phi(L(x, x))$ defines a prehilbert seminorm on E.

With the help of the previous remark, we can now obtain the following Pietsch's factorization theorem in the setting of real JB*-triples.

Theorem 4.3. *Let* T *be a linear operator from a real JB*-triple* E *with values in a real Banach space* Y. *The following assertions are equivalent.*

1. T *is 2-JB*-triple-summing.*

2. *There is a state* $\Psi \in D(BL(E), I_E)$ *and a positive constant* $C(T)$ *such that*
$$\|T(x)\| \leq C(T)|||x|||_\Psi$$
for every $x \in E$.

3. *There are norm-one functionals* φ_1, φ_2 *in* E^* *and a positive constant* $C(T)'$ *such that*
$$\|T(x)\| \leq C(T)'\|x\|_{\varphi_1,\varphi_2}$$
for all $x \in E$.

Proof. 1 $\Rightarrow$ 2.− By Remark 4.2 above, we see that T can be extended to a complex linear operator $\widehat{T} : \widehat{E} \to \tilde{Y}$ which is also 2-JB*-triple summing, where $\tilde{Y}$ denotes the complexification of Y defined in Remark 4.2. Now by Theorem 3.6 there exists a state $\Phi \in D(BL(\widehat{E}), I_{\widehat{E}})$ and a positive constant $C(\widehat{T})$ such that

$$\|T(x)\| \leq C(\widehat{T})\||x\||_\Phi \leq 2\sqrt{2C_2(T)}\||x\||_\Phi$$

for every $x \in E$. By [19, Corollary 1.7] there exists $\Psi \in D(BL(E), I_E)$ such that

$$\||x\||_\Phi = \||x\||_\Psi$$

for all $x \in E$. Therefore

$$\|T(x)\| \leq 2\sqrt{2C_2(T)}\||x\||_\Psi$$

for every $x \in E$.

The rest of the proof runs as in Theorem 3.6.

□

The next lemma can be derived form Theorem 4.3 above as Lemma 3.7 was derived from Theorem 3.6.

Lemma 4.4. *Let $T : E \to Y$ be a 2-JB*-triple-summing operator from a real JB*-triple to a Banach space. Then there are norm-one functionals φ_1, φ_2 in E^* and a positive constant $C(T)$ such that*

$$\|T^{**}(\alpha)\| \leq C(T)\|\alpha\|_{\varphi_1,\varphi_2}$$

for all $\alpha \in E^{**}$. In particular T^{**} is 2-JB-triple-summing.*

Our last goal is to obtain a weak*-version of Theorem 4.3 above. The next remark play a fundamental role in the proof of such result.

Remark 4.5. *Let $T : W \to Y^*$ be a 2-JB*-triple-summing and weak*-continuous operator form a real JBW*-triple to a dual Banach space. Let us denote by $\widehat{W}$ and τ the complexification of W and the canonical conjugation τ on $\widehat{W}$, respectively. We define*

$$\phi : \widehat{W}^* \to \widehat{W}^*$$

by

$$\phi(f)(z) = \overline{f(\tau(z))}.$$

From [13] we can assure that ϕ *is a conjugation (conjugate-linear isometry of period 2) on* $\widehat{W}^*$. *Furthermore the map*

$$(\widehat{W}^*)^{\phi} := \{f \in \widehat{W}^* : \phi(f) = f\} \to (\widehat{W}^{\tau})^*$$

$$f \mapsto f|_W$$

is an isometric bijection. In the same way, the predual W_* *of* W *can be identified with* $(\widehat{W}_*)^{\phi} := \{f \in \widehat{W}_* : \phi(f) = f\}$. *The construction can be realized one more time to get a conjugation* $\widehat{\phi}$ *on* $\widehat{W}^{**}$ *such that*

$$W^{**} \cong (\widehat{W}^{**})^{\widehat{\phi}}.$$

Since T *is weak*-continuous, there is a bounded linear operator* $R : W_* \to Y$ *such that* $R^* = T$. *Let* $\widetilde{Y}$ *denote the complexification of* Y *defined in Remark 4.2 and* $\widetilde{R} : \widehat{W}_* \to \widetilde{Y}$ *the complex linear extension of* R. *Then* $\widetilde{T} := (\widetilde{R})^* : \widehat{W} \to (\widetilde{Y})^*$ *is a weak*-continuous operator extending* T *to* $\widehat{W}$ *and verifying* $\|\widetilde{T}\| = \|\widetilde{R}\| \leq 2\|R\| = 2\|T\|$. *Now we can repeat the same arguments given in Remark 4.2 to assure that* $\widetilde{T}$ *is 2-JB*-triple-summing (and* $C_2(\widetilde{T}) \leq 2C_2(T)$*).*

We can now state the weak*-version of Theorem 4.3.

Theorem 4.6. *Let* T *be a weak*-continuous linear operator from a real JBW*-triple* W *with values in a real Banach space* Y^*. *The following assertions are equivalent.*

1. T *is 2-JB*-triple-summing.*

2. *There are norm-one functionals* φ_1, φ_2 *in* W_* *and a positive constant* $C(T)$ *such that*

   $$\|T(x)\| \leq C(T)\|x\|_{\varphi_1,\varphi_2}$$

   for all $x \in W$.

Proof. 1 ⇒ 2.– By Remark 4.5 above, we see that T can be extended to a weak*-continuous operator $\widetilde{T} : \widehat{W} \to (\tilde{Y})^*$ which is also 2-JB*-triple summing, where $\tilde{Y}$ denotes the complexification of Y defined in Remark 4.2. Now by Theorem 3.5 there are norm-one functionals ψ_1, ψ_2 in $\widehat{W}_*$ and a positive constant $C(\tilde{T})$ such that

$$\|T(x)\| \leq C(\widehat{T}) \ \|x\|_{\psi_1,\psi_2} \leq 2\sqrt{2C_2(T)} \ \|x\|_{\psi_1,\psi_2} \tag{10}$$

for all $x \in W$.

Let $e_1, e_2 \in S_{\widehat{W}}$ with $\psi_1(e_1) = \psi_2(e_2) = 1$. The map $(x, y) \mapsto (x|y) := \Re e(\psi_1 \{x, y, e_1\} + \psi_2 \{x, y, e_2\})$ is a positive bilinear form on W. If we denote $N := \{x \in W : (x|x) = 0\}$, the quotient W/N can be completed to a a Hilbert space, which is denoted by H. The natural quotient map of W on H will be denoted by J_{ψ_1,ψ_2}. We note that, by the same arguments given in the proof of [22, Corollary 1], it may be concluded that J_{ψ_1,ψ_2} is weak*-continuous. Now By [18, Theorem 5] it follows that there exist norm-one functionals $\varphi_1, \varphi_2 \in S_{W_*}$ such that

$$\|J_{\psi_1,\psi_2}(x)\|^2 = \Re e(\psi_1 \{x, x, e_1\} + \psi_2 \{x, x, e_2\}) = \|x\|^2_{\psi_1,\psi_2} \leq 6^2 \ \|x\|^2_{\varphi_1,\varphi_2}$$

for all $x \in W$. Therefore, by (10), we conclude that

$$\|T(x)\| \leq 12\sqrt{2C_2(T)} \ \|x\|_{\varphi_1,\varphi_2}$$

for all $x \in W$.

The implication $2 \Rightarrow 1.-$ follows as in Theorem 3.5.

□

Remark 4.7. *Let $T : \mathcal{E} \to Y$ be a 2-summing operator from a real or complex JB*-triple to a Banach space. Let $\varphi \in S_{\mathcal{E}^*}$ and $z \in S_{\mathcal{E}^{**}}$ satisfying $\varphi(z) = 1$. By [2, Proof of Theorem 3.2] we have*

$$|\varphi(x)| \leq \|x\|_\varphi = (\varphi(L(x, x)z))^{\frac{1}{2}}$$

for all x in $\mathcal{E}$, and hence

$$\sum_{k=1}^{n} \|T(x_k)\|^2 \leq C_2(T)^2 \sup \left\{ \sum_{k=1}^{n} f(L(x, x)z) : f \in S_{\mathcal{E}^*}, z \in S_{\mathcal{E}}, f(z) = 1 \right\}$$

$$\leq C_2(T)^2 \Big\| \sum_{i=1}^{n} L(x_i, x_i) \Big\|,$$

for every finite sequence $(x_1, \ldots, x_n) \subseteq \mathcal{E}$. Therefore every 2-summing operator from a real or complex JB-triple to a Banach space is 2-JB*-triple-summing.*

Corollary 4.8. *Let T be a 2-summing operator from a real or complex JB*-triple E to a Banach space. Then there are norm-one functionals φ_1, φ_2 in E^* and a positive constant $C(T)$ such that*

$$\|T(x)\| \leq C(T)\|x\|_{\varphi_1,\varphi_2}$$

for all $x \in E$.

Let X and Y be Banach spaces. We recall that an operator $T : X \to Y$ is said to be of *cotype* q $(2 \leq q < \infty)$, if there is a constant C such that for any $\{x_1, \ldots, x_n\} \subseteq X$ the inequality

$$(\sum_{j=1}^{n} \|T(x_j\|^q)^{\frac{1}{q}} \leq C \left(\int_D \| \sum_{j=1}^{n} \varepsilon_j x_j \|^2 d(\mu) \right)^{\frac{1}{2}}$$

holds, where $\varepsilon_j \in \{-1, 1\}$; $D = \{-1, 1\}^{\mathbb{N}}$ and μ is the uniform probability measure on D. A Banach space X is said to be of cotype q if I_X is of cotype q. By [21, page 120], we know that if X is a Banach space of cotype q then I_X is $(q, 1)-$summing, i. e., there is a constant C such that, for all finite sequences (x_i) in X, we have

$$\left(\sum_{i=1}^{n} \|x_i\|^q\right)^{\frac{1}{q}} \leq C \ \sup\{\sum_{i=1}^{n} |\xi(x_i)| : \xi \in X^*, \|\xi\| \leq 1\}.$$

In general it can not be expected that if Y is a Banach space of cotype 2 then I_Y could be 2-summing. However, as we are showing in what follows, if Y is a Banach space of cotype 2 then every bounded linear operator from a real or complex JB*-triple to Y is always 2-JB*-triple-summing. Indeed, in [7, Theorem 12] C-H. Chu, B. Iochum and G. Loupias show that if $T : \mathcal{E} \to Y$ is a bounded linear operator from a JB*-triple to a Banach space of cotype 2, then there are norm-one functionals φ_1, φ_2 in $\mathcal{E}^*$ and a positive constant $C(Y)$ (depending only on Y) such that

$$\|T(x)\| \leq C(Y)\|T\| \ \|x\|_{\varphi_1,\varphi_2} \tag{11}$$

for all $x \in \mathcal{E}$. Therefore, we conclude by Theorem 3.6 that T is 2-JB*-triple-summing. We have thus proved the following corollary.

Corollary 4.9. *Every bounded linear operator from a real or complex JB*-triple to a Banach space of cotype 2 is 2-JB*-triple-summing.*

Remark 4.10. *It is worth mentioning that in [7, Theorem 12] the authors affirm that if* $T : \mathcal{E} \to Y$ *is a bounded linear operator from a JB*-triple to a Banach space of cotype 2, then there exists a norm-one functional* φ *in* $\mathcal{E}^*$ *and a positive constant* $C(Y)$ *(depending only on* Y*) such that*

$$\|T(x)\| \leq C(Y)\|T\| \ \|x\|_\varphi$$

for all $x \in \mathcal{E}$*. In the proof of this theorem, the result [7, Proposition 4] ([1, Theorem 1.3]) play a fundamental role. Since, as we have mentioned before, the proof of the last result contains some subtle difficulties (compare [17, 18]), the original setting of [7, Theorem 12] is only a conjecture. However, when in the Chu-Iochum-Loupias proof, [18, Theorem 3] (see also [19, Corollary 1.11]) replaces [7, Proposition 4] we obtain the statement in* (11)*.*

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